7

Magnetic circuits

At the end of this chapter you should be able to:

- describe the magnetic field around a permanent magnet
- state the laws of magnetic attraction and repulsion for two magnets in close proximity
- define magnetic flux, Φ , and magnetic flux density, B, and state their units
- perform simple calculations involving $B = \Phi/A$
- define magnetomotive force, $F_{\rm m}$, and magnetic field strength, H, and state their units
- perform simple calculations involving $F_{\rm m} = NI$ and H = NI/l
- define permeability, distinguishing between μ_0 , μ_r and μ
- understand the B-H curves for different magnetic materials
- appreciate typical values of μ_r
- perform calculations involving $B = \mu_0 \mu_r H$
- define reluctance, *S*, and state its units
- perform calculations involving

$$S = \frac{\text{m.m.f.}}{\Phi} = \frac{l}{\mu_0 \mu_{rd}}$$

- perform calculations on composite series magnetic circuits
- compare electrical and magnetic quantities
- appreciate how a hysteresis loop is obtained and that hysteresis loss is proportional to its area

7.1 Magnetic fields

A **permanent magnet** is a piece of ferromagnetic material (such as iron, nickel or cobalt) which has properties of attracting other pieces of these materials. A permanent magnet will position itself in a north and south direction when freely suspended. The north-seeking end of the magnet is called the **north pole**, **N**, and the south-seeking end the **south pole**, **S**.

The area around a magnet is called the **magnetic** field and it is in this area that the effects of the

magnetic force produced by the magnet can be detected. A magnetic field cannot be seen, felt, smelt or heard and therefore is difficult to represent. Michael Faraday suggested that the magnetic field could be represented pictorially, by imagining the field to consist of **lines of magnetic flux**, which enables investigation of the distribution and density of the field to be carried out.

The distribution of a magnetic field can be investigated by using some iron filings. A bar magnet is placed on a flat surface covered by, say, cardboard, upon which is sprinkled some iron filings. If the cardboard is gently tapped the filings will assume a pattern similar to that shown in Fig. 7.1. If a number of magnets of different strength are used, it is found that the stronger the field the closer are the lines of magnetic flux and vice versa. Thus a magnetic field has the property of exerting a force, demonstrated in this case by causing the iron filings to move into the pattern shown. The strength of the magnetic field decreases as we move away from the magnet. It should be realized, of course, that the magnetic field is three dimensional in its effect, and not acting in one plane as appears to be the case in this experiment.





If a compass is placed in the magnetic field in various positions, the direction of the lines of flux may be determined by noting the direction of the compass pointer. The direction of a magnetic field at any point is taken as that in which the north-seeking pole of a compass needle points when suspended in the field. The direction of a line of flux is from the north pole to the south pole on the outside of the magnet and is then assumed to continue through the magnet back to the point at which it emerged at the north pole. Thus such lines of flux always form complete closed loops or paths, they never intersect and always have a definite direction.

The laws of magnetic attraction and repulsion can be demonstrated by using two bar magnets. In Fig. 7.2(a), **with unlike poles adjacent, attraction takes place**. Lines of flux are imagined to contract and the magnets try to pull together. The magnetic field is strongest in between the two magnets, shown by the lines of flux being close together. In Fig. 7.2(b), **with similar poles adjacent (i.e. two north poles), repulsion occurs**, i.e. the two north poles try to push each other apart, since magnetic flux lines running side by side in the same direction repel.

7.2 Magnetic flux and flux density

Magnetic flux is the amount of magnetic field (or the number of lines of force) produced by a



Figure 7.2

magnetic source. The symbol for magnetic flux is Φ (Greek letter 'phi'). The unit of magnetic flux is the **weber**, **Wb**

Magnetic flux density is the amount of flux passing through a defined area that is perpendicular to the direction of the flux:

Magnetic flux density =
$$\frac{\text{magnetic flux}}{\text{area}}$$

The symbol for magnetic flux density is *B*. The unit of magnetic flux density is the tesla, *T*, where $1 T = 1 \text{ Wb/m}^2$. Hence

$$B = \frac{\Phi}{A}$$
 tesla

where $A(m^2)$ is the area

Problem 1. A magnetic pole face has a rectangular section having dimensions 200 mm by 100 mm. If the total flux emerging from the pole is 150μ Wb, calculate the flux density.

Flux $\Phi = 150 \,\mu\text{Wb} = 150 \times 10^{-6} \,\text{Wb}$ Cross sectional area $A = 200 \times 100 = 20\,000 \,\text{mm}^2 = 20\,000 \times 10^{-6} \,\text{m}^2$.

Flux density,
$$B = \frac{\Phi}{A} = \frac{150 \times 10^{-6}}{20\,000 \times 10^{-6}}$$

= **0.0075 T** or **7.5 mT**

Problem 2. The maximum working flux density of a lifting electromagnet is 1.8 T and the effective area of a pole face is circular in cross-section. If the total magnetic flux produced is 353 mWb, determine the radius of the pole face.

Flux density B = 1.8 T and flux $\Phi = 353$ mWb = 353×10^{-3} Wb.

Since $B = \Phi/A$, cross-sectional area $A = \Phi/B$

$$=\frac{353\times10^{-3}}{1.8}\,\mathrm{m}^2=0.1961\,\mathrm{m}^2$$

=

The pole face is circular, hence area = πr^2 , where *r* is the radius. Hence $\pi r^2 = 0.1961$ from which, $r^2 = 0.1961/\pi$ and radius $r = \sqrt{(0.1961/\pi)} = 0.250$ m i.e. **the radius of the pole face is 250 mm**.

7.3 Magnetomotive force and magnetic field strength

Magnetomotive force (m.m.f.) is the cause of the existence of a magnetic flux in a magnetic circuit,

m.m.f.
$$F_{\rm m} = NI$$
 amperes

where N is the number of conductors (or turns) and I is the current in amperes. The unit of mmf is sometimes expressed as 'ampere-turns'. However since 'turns' have no dimensions, the S.I. unit of m.m.f. is the ampere.

Magnetic field strength (or magnetising force),

$$H = \frac{NI}{l}$$
 ampere per metre

where l is the mean length of the flux path in metres. Thus

m.m.f.
$$= NI = Hl$$
 amperes

Problem 3. A magnetising force of 8000 A/m is applied to a circular magnetic circuit of mean diameter 30 cm by passing a current through a coil wound on the circuit. If the coil is uniformly wound around the circuit and has 750 turns, find the current in the coil.

H = 8000 A/m, $l = \pi d = \pi \times 30 \times 10^{-2}$ m and N = 750 turns. Since H = NI/l, then

$$I = \frac{Hl}{N} = \frac{8000 \times \pi \times 30 \times 10^{-2}}{750}$$

Thus, current $I = 10.05 \,\mathrm{A}$

Now try the following exercise

Exercise 31 Further problems on magnetic circuits

- 1 What is the flux density in a magnetic field of cross-sectional area 20 cm² having a flux of 3 mWb? [1.5 T]
- 2 Determine the total flux emerging from a magnetic pole face having dimensions 5 cm by 6 cm, if the flux density is 0.9 T [2.7 mWb]
- 3 The maximum working flux density of a lifting electromagnet is 1.9 T and the effective area of a pole face is circular in cross-section. If the total magnetic flux produced is 611 mWb determine the radius of the pole face. [32 cm]
- 4 An electromagnet of square cross-section produces a flux density of 0.45 T. If the magnetic flux is 720μ Wb find the dimensions of the electromagnet cross-section. [4 cm by 4 cm]
- 5 Find the magnetic field strength applied to a magnetic circuit of mean length 50 cm when a coil of 400 turns is applied to it carrying a current of 1.2 A [960 A/m]
- 6 A solenoid 20 cm long is wound with 500 turns of wire. Find the current required to establish a magnetising force of 2500 A/m inside the solenoid. [1 A]
- 7 A magnetic field strength of 5000 A/m is applied to a circular magnetic circuit of mean diameter 250 mm. If the coil has 500 turns find the current in the coil. [7.85 A]

7.4 Permeability and B–H curves

For air, or any non-magnetic medium, the ratio of magnetic flux density to magnetising force is a constant, i.e. B/H = a constant. This constant is

 μ_0 , the **permeability of free space** (or the magnetic space constant) and is equal to $4\pi \times 10^{-7}$ H/m, i.e. **for air, or any non-magnetic medium**, the ratio

$$rac{B}{H}=\mu_0$$

(Although all non-magnetic materials, including air, exhibit slight magnetic properties, these can effectively be neglected.)

For all media other than free space,

$$rac{B}{H}=\mu_0\mu_{
m r}$$

where u_r is the relative permeability, and is defined as



 $\mu_{\rm r}$ varies with the type of magnetic material and, since it is a ratio of flux densities, it has no unit. From its definition, $\mu_{\rm r}$ for a vacuum is 1. $\mu_0\mu_{\rm r} = \mu$, called the **absolute permeability**

By plotting measured values of flux density B against magnetic field strength H, a **magnetisation curve** (or **B-H curve**) is produced. For nonmagnetic materials this is a straight line. Typical curves for four magnetic materials are shown in Fig. 7.3

The **relative permeability** of a ferromagnetic material is proportional to the slope of the B–H curve and thus varies with the magnetic field strength. The approximate range of values of relative permeability μ_r for some common magnetic materials are:

Cast iron	$\mu_{\rm r} = 100 - 250$
Mild steel	$\mu_{\rm r} = 200 - 800$
Silicon iron	$\mu_{\rm r} = 1000 - 5000$
Cast steel	$\mu_{\rm r} = 300 - 900$
Mumetal	$\mu_{\rm r} = 200 - 5000$
Stalloy	$\mu_{\rm r} = 500 - 6000$

Problem 4. A flux density of 1.2 T is produced in a piece of cast steel by a magnetising force of 1250 A/m. Find the relative permeability of the steel under these conditions.



Figure 7.3

For a magnetic material: $B = \mu_0 \mu_r H$

i.e.
$$\mu_{\rm r} = \frac{B}{\mu_0 H} = \frac{1.2}{(4\pi \times 10^{-7})(1250)} = 764$$

Problem 5. Determine the magnetic field strength and the m.m.f. required to produce a flux density of 0.25 T in an air gap of length 12 mm.

For air: $B = \mu_0 H$ (since $\mu_r = 1$) Magnetic field strength,

$$H = \frac{B}{\mu_0} = \frac{0.25}{4\pi \times 10^{-7}} = 198\,940\,\text{A/m}$$

$$\mathbf{m}$$
. \mathbf{m} . \mathbf{f} . = $Hl = 198\,940 \times 12 \times 10^{-3} = \mathbf{2387}\,\mathbf{A}$

Problem 6. A coil of 300 turns is wound uniformly on a ring of non-magnetic material. The ring has a mean circumference of 40 cm and a uniform cross-sectional area of 4 cm^2 . If the current in the coil is 5 A, calculate (a) the magnetic field strength, (b) the flux density and (c) the total magnetic flux in the ring. (a) Magnetic field strength

$$H = \frac{NI}{l} = \frac{300 \times 5}{40 \times 10^{-2}} = 3750 \,\text{A/m}$$

(b) For a non-magnetic material $\mu_r = 1$, thus flux density $B = \mu_0 H$

i.e.
$$B = 4\pi \times 10^{-7} \times 3750$$

$$= 4.712 \,\mathrm{mT}$$

(c) Flux
$$\Phi = BA = (4.712 \times 10^{-3})(4 \times 10^{-4})$$

 $= 1.885 \,\mu Wb$

Problem 7. An iron ring of mean diameter 10 cm is uniformly wound with 2000 turns of wire. When a current of 0.25 A is passed through the coil a flux density of 0.4 T is set up in the iron. Find (a) the magnetising force and (b) the relative permeability of the iron under these conditions.

 $l = \pi d = \pi \times 10 \text{ cm} = \pi \times 10 \times 10^{-2} \text{ m},$ N = 2000 turns, I = 0.25 A and B = 0.4 T

(a)
$$H = \frac{NI}{l} = \frac{2000 \times 0.25}{\pi \times 10 \times 10^{-2}}$$

= **1592 A/m**
(b) $B = \mu_0 \mu_r H$, hence μ_r

$$=\frac{B}{\mu_0 H}=\frac{0.4}{(4\pi\times10^{-7})(1592)}=200$$

Problem 8. A uniform ring of cast iron has a cross-sectional area of 10 cm^2 and a mean circumference of 20 cm. Determine the m.m.f. necessary to produce a flux of 0.3 mWb in the ring. The magnetisation curve for cast iron is shown on page 71.

 $A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{m}^2$, l = 20 cm = 0.2 mand $\Phi = 0.3 \times 10^{-3} \text{ Wb}$.

Flux density
$$B = \frac{\Phi}{A} = \frac{0.3 \times 10^{-3}}{10 \times 10^{-4}} = 0.3 \text{ T}$$

From the magnetisation curve for cast iron on page 71, when B = 0.3 T, H = 1000 A/m, hence **m.m.f.** = $Hl = 1000 \times 0.2 = 200$ A

A tabular method could have been used in this problem. Such a solution is shown below in Table 1.

Problem 9. From the magnetisation curve for cast iron, shown on page 71, derive the curve of μ_r against *H*.

 $B = \mu_0 \mu_r H$, hence

$$\mu_{\rm r} = \frac{B}{\mu_0 H} = \frac{1}{\mu_0} \times \frac{B}{H}$$
$$= \frac{10^7}{4\pi} \times \frac{B}{H}$$

A number of co-ordinates are selected from the B–H curve and μ_r is calculated for each as shown in Table 2.

Table 1													
Part of circuit	Material Cast iron		Φ (Wb) 0.3×10^{-3}		$A(m^2)$ 10×10^{-4}		$B = \frac{\Phi}{A}(T)$ 0.3		H from graph	<i>l</i> (<i>m</i>)) m <i>H</i>	m.m.f. = Hl(A) 200	
Ring									1000	0.2	20		
Table 2													
B(T)		0.04	0.13	0.17	0.30	0.41	0.49	0.60	0.68	0.73	0.76	0.79	
H(A/m)		200	400	500	1000	1500	2000	3000	4000	5000	6000	7000	
$\mu_{\rm r} = \frac{10^7}{4\pi}$	$\times \frac{B}{H}$	159	259	271	239	218	195	159	135	116	101	90	

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 $\mu_{\rm r}$ is plotted against *H* as shown in Fig. 7.4. The curve demonstrates the change that occurs in the relative permeability as the magnetising force increases.





Now try the following exercise

Exercise 32 Further problems on magnetic circuits

(Where appropriate, assume $\mu_0 = 4\pi \times 10^{-7}$ H/m)

- 1 Find the magnetic field strength and the magnetomotive force needed to produce a flux density of 0.33 T in an air-gap of length 15 mm. [(a) 262 600 A/m (b) 3939 A]
- 2 An air-gap between two pole pieces is 20 mm in length and the area of the flux path across the gap is 5 cm^2 . If the flux required in the air-gap is 0.75 mWb find the m.m.f. necessary. [23 870 A]
- 3 (a) Determine the flux density produced in an air-cored solenoid due to a uniform magnetic field strength of 8000 A/m (b) Iron having a relative permeability of 150 at 8000 A/m is inserted into the solenoid of part (a). Find the flux density now in the solenoid.

[(a) 10.05 mT (b) 1.508 T]

4 Find the relative permeability of a material if the absolute permeability is 4.084×10^{-4} H/m. [325]

- 5 Find the relative permeability of a piece of silicon iron if a flux density of 1.3 T is produced by a magnetic field strength of 700 A/m [1478]
- 6 A steel ring of mean diameter 120 mm is uniformly wound with 1500 turns of wire. When a current of 0.30 A is passed through the coil a flux density of 1.5 T is set up in the steel. Find the relative permeability of the steel under these conditions. [1000]
- 7 A uniform ring of cast steel has a crosssectional area of 5 cm^2 and a mean circumference of 15 cm. Find the current required in a coil of 1200 turns wound on the ring to produce a flux of 0.8 mWb. (Use the magnetisation curve for cast steel shown on page 71) [0.60 A]
- 8 (a) A uniform mild steel ring has a diameter of 50 mm and a cross-sectional area of 1 cm^2 . Determine the m.m.f. necessary to produce a flux of 50 µWb in the ring. (Use the B–H curve for mild steel shown on page 71) (b) If a coil of 440 turns is wound uniformly around the ring in Part (a) what current would be required to produce the flux?

[(a) 110 A (b) 0.25 A]

9 From the magnetisation curve for mild steel shown on page 71, derive the curve of relative permeability against magnetic field strength. From your graph determine (a) the value of μ_r when the magnetic field strength is 1200 A/m, and (b) the value of the magnetic field strength when μ_r is 500 [(a) 590–600 (b) 2000]

7.5 Reluctance

Reluctance S (or R_M) is the 'magnetic resistance' of a magnetic circuit to the presence of magnetic flux. **Reluctance**,

$$S = \frac{F_{\rm M}}{\Phi} = \frac{NI}{\Phi} = \frac{Hl}{BA} = \frac{l}{(B/H)A} = \frac{l}{\mu_0\mu_{\rm r}A}$$

The unit of reluctance is 1/H (or H^{-1}) or A/Wb.

Ferromagnetic materials have a low reluctance and can be used as **magnetic screens** to prevent magnetic fields affecting materials within the screen.

9

Electromagnetic induction

At the end of this chapter you should be able to:

- understand how an e.m.f. may be induced in a conductor
- state Faraday's laws of electromagnetic induction
- state Lenz's law
- use Fleming's right-hand rule for relative directions
- appreciate that the induced e.m.f., E = Blv or $E = Blv \sin \theta$
- calculate induced e.m.f. given B, l, v and θ and determine relative directions
- define inductance L and state its unit
- define mutual inductance
- appreciate that emf

$$E = -N\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

- calculate induced e.m.f. given N, t, L, change of flux or change of current
- appreciate factors which affect the inductance of an inductor
- draw the circuit diagram symbols for inductors
- calculate the energy stored in an inductor using $W = \frac{1}{2}LI^2$ joules
- calculate inductance L of a coil, given $L = N\Phi/I$
- calculate mutual inductance using $E_2 = -M(dI_1/dt)$

9.1 Introduction to electromagnetic induction

When a conductor is moved across a magnetic field so as to cut through the lines of force (or flux), an electromotive force (e.m.f.) is produced in the conductor. If the conductor forms part of a closed circuit then the e.m.f. produced causes an electric current to flow round the circuit. Hence an e.m.f. (and thus current) is 'induced' in the conductor as a result of its movement across the magnetic field. This effect is known as 'electromagnetic induction'.

Figure 9.1 (a) shows a coil of wire connected to a centre-zero galvanometer, which is a sensitive ammeter with the zero-current position in the centre of the scale.

(a) When the magnet is moved at constant speed towards the coil (Fig. 9.1(a)), a deflection is noted on the galvanometer showing that a current has been produced in the coil.



Figure 9.1

- (b) When the magnet is moved at the same speed as in (a) but away from the coil the same deflection is noted but is in the opposite direction (see Fig. 9.1(b))
- (c) When the magnet is held stationary, even within the coil, no deflection is recorded.
- (d) When the coil is moved at the same speed as in (a) and the magnet held stationary the same galvanometer deflection is noted.
- (e) When the relative speed is, say, doubled, the galvanometer deflection is doubled.
- (f) When a stronger magnet is used, a greater galvanometer deflection is noted.
- (g) When the number of turns of wire of the coil is increased, a greater galvanometer deflection is noted.

Figure 9.1(c) shows the magnetic field associated with the magnet. As the magnet is moved towards the coil, the magnetic flux of the magnet moves across, or cuts, the coil. It is the relative movement of the magnetic flux and the coil that causes an e.m.f. and thus current, to be induced in the coil. This effect is known as electromagnetic induction. The laws of electromagnetic induction stated in section 9.2 evolved from experiments such as those described above.

9.2 Laws of electromagnetic induction

Faraday's laws of electromagnetic induction state:

- *(i)* An induced e.m.f. is set up whenever the magnetic field linking that circuit changes.
- (ii) The magnitude of the induced e.m.f. in any circuit is proportional to the rate of change of the magnetic flux linking the circuit.

Lenz's law states:

The direction of an induced e.m.f. is always such that it tends to set up a current opposing the motion or the change of flux responsible for inducing that e.m.f.

An alternative method to Lenz's law of determining relative directions is given by **Fleming's <u>Right-hand rule**</u> (often called the gene<u>R</u>ator rule) which states:

Let the thumb, first finger and second finger of the right hand be extended such that they are all at right angles to each other (as shown in Fig. 9.2). If the first finger points in the direction of the magnetic field and the thumb points in the direction of motion of the conductor relative to the magnetic field, then the second finger will point in the direction of the induced e.m.f. Summarising:

First finger - Field

ThuMb - Motion

SEcond finger - E.m.f.



Figure 9.2

In a generator, conductors forming an electric circuit are made to move through a magnetic field. By Faraday's law an e.m.f. is induced in the conductors and thus a source of e.m.f. is created. A generator converts mechanical energy into electrical energy. (The action of a simple a.c. generator is described in Chapter 14).

The induced e.m.f. E set up between the ends of the conductor shown in Fig. 9.3 is given by:



Figure 9.3

where B, the flux density, is measured in teslas, l, the length of conductor in the magnetic field, is measured in metres, and v, the conductor velocity, is measured in metres per second.

If the conductor moves at an angle θ° to the magnetic field (instead of at 90° as assumed above) then

$$E = Blv \sin \theta$$
 volts

Problem 1. A conductor 300 mm long moves at a uniform speed of 4 m/s at right-angles to a uniform magnetic field of flux density 1.25 T. Determine the current flowing in the conductor when (a) its ends are open-circuited, (b) its ends are connected to a load of 20 Ω resistance.

When a conductor moves in a magnetic field it will have an e.m.f. induced in it but this e.m.f. can only produce a current if there is a closed circuit. Induced e.m.f.

$$E = Blv = (1.25) \left(\frac{300}{1000}\right) (4) = 1.5 \,\mathrm{V}$$

- (a) If the ends of the conductor are open circuited **no current will flow** even though 1.5 V has been induced.
- (b) From Ohm's law,

$$I = \frac{E}{R} = \frac{1.5}{20} = 0.075 \,\mathrm{A} \text{ or } 75 \,\mathrm{mA}$$

Problem 2. At what velocity must a conductor 75 mm long cut a magnetic field of flux density 0.6 T if an e.m.f. of 9 V is to be induced in it? Assume the conductor, the field and the direction of motion are mutually perpendicular.

Induced e.m.f. E = Blv, hence velocity v = E/BlThus

$$v = \frac{9}{(0.6)(75 \times 10^{-3})}$$
$$= \frac{9 \times 10^3}{0.6 \times 75}$$
$$= 200 \text{ m/s}$$

Problem 3. A conductor moves with a velocity of 15 m/s at an angle of (a) 90° (b) 60° and (c) 30° to a magnetic field produced between two square-faced poles of side length 2 cm. If the flux leaving a pole face is 5μ Wb, find the magnitude of the induced e.m.f. in each case.

v = 15 m/s, length of conductor in magnetic field, $l = 2 \text{ cm} = 0.02 \text{ m}, A = 2 \times 2 \text{ cm}^2 = 4 \times 10^{-4} \text{m}^2$ and $\Phi = 5 \times 10^{-6}$ Wb

(a)
$$E_{90} = Blv \sin 90^{\circ}$$

 $= \left(\frac{\Phi}{A}\right) lv \sin 90^{\circ}$
 $= \left(\frac{5 \times 10^{-6}}{4 \times 10^{-4}}\right) (0.02) (15) (1)$
 $= 3.75 \,\mathrm{mV}$

(b)
$$E_{60} = Blv \sin 60^\circ = E_{90} \sin 60^\circ$$

= 3.75 sin 60° = **3.25 mV**

(c)
$$E_{30} = Blv \sin 30^\circ = E_{90} \sin 30^\circ$$

= 3.75 sin 30° = **1.875 mV**

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Problem 4. The wing span of a metal aeroplane is 36 m. If the aeroplane is flying at 400 km/h, determine the e.m.f. induced between its wing tips. Assume the vertical component of the earth's magnetic field is $40 \mu \text{T}$.

Induced e.m.f. across wing tips, E = Blv $B = 40 \,\mu\text{T} = 40 \times 10^{-6} \text{ T}, l = 36 \text{ m}$ and

$$v = 400 \frac{\text{km}}{\text{h}} \times 1000 \frac{\text{m}}{\text{km}} \times \frac{1 \text{ h}}{60 \times 60 \text{ s}}$$
$$= \frac{(400)(1000)}{3600}$$
$$= \frac{4000}{36} \text{ m/s}$$

Hence

$$E = Blv = (40 \times 10^{-6})(36) \left(\frac{4000}{36}\right)$$
$$= 0.16 \text{ V}$$

Force on conductor (a) (b) (c) Notion of conductor (c) Notion of conductor (c) Notion of conductor Notion Not

represents the generation of e.m.f's. Determine (i) the direction in which the conductor has to be moved in Fig. 9.4(a), (ii) the direction of the induced e.m.f. in Fig. 9.4(b), (iii) the polarity of the magnetic system in Fig. 9.4(c)

Problem 5. The diagrams shown in Fig. 9.4



The direction of the e.m.f., and thus the current due to the e.m.f. may be obtained by either Lenz's law or Fleming's <u>Right-hand</u> rule (i.e. Gene<u>R</u>ator rule).

(i) Using Lenz's law: The field due to the magnet and the field due to the current-carrying conductor are shown in Fig. 9.5(a) and are



seen to reinforce to the left of the conductor. Hence the force on the conductor is to the right. However Lenz's law states that the direction of the induced e.m.f. is always such as to oppose the effect producing it. **Thus the conductor will have to be moved to the left**.

(ii) Using Fleming's right-hand rule:

First finger - Field,

i.e. $N \rightarrow S$, or right to left;

Thu<u>M</u>b - <u>M</u>otion, i.e. upwards;

SEcond finger - E.m.f.

i.e. towards the viewer or out of the paper, as shown in Fig. 9.5(b)

(iii) The polarity of the magnetic system of Fig. 9.4(c) is shown in Fig. 9.5(c) and is obtained using Fleming's right-hand rule. Now try the following exercise

Exercise 41 Further problems on induced e.m.f.

- 1 A conductor of length 15 cm is moved at 750 mm/s at right-angles to a uniform flux density of 1.2 T. Determine the e.m.f. induced in the conductor. [0.135 V]
- 2 Find the speed that a conductor of length 120 mm must be moved at right angles to a magnetic field of flux density 0.6 T to induce in it an e.m.f. of 1.8 V [25 m/s]
- 3 A 25 cm long conductor moves at a uniform speed of 8 m/s through a uniform magnetic field of flux density 1.2 T. Determine the current flowing in the conductor when (a) its ends are open-circuited, (b) its ends are connected to a load of 15 ohms resistance.

[(a) 0 (b) 0.16 A]

- 4 A straight conductor 500 mm long is moved with constant velocity at right angles both to its length and to a uniform magnetic field. Given that the e.m.f. induced in the conductor is 2.5 V and the velocity is 5 m/s, calculate the flux density of the magnetic field. If the conductor forms part of a closed circuit of total resistance 5 ohms, calculate the force on the conductor. [1 T, 0.25 N]
- 5 A car is travelling at 80 km/h. Assuming the back axle of the car is 1.76 m in length and the vertical component of the earth's magnetic field is 40μ T, find the e.m.f. generated in the axle due to motion. [1.56 mV]
- 6 A conductor moves with a velocity of 20 m/s at an angle of (a) 90° (b) 45° (c) 30°, to a magnetic field produced between two square-faced poles of side length 2.5 cm. If the flux on the pole face is 60 mWb, find the magnitude of the induced e.m.f. in each case.

[(a) 48 V (b) 33.9 V (c) 24 V]

9.3 Inductance

Inductance is the name given to the property of a circuit whereby there is an e.m.f. induced into the circuit by the change of flux linkages produced by a current change.

When the e.m.f. is induced in the same circuit as that in which the current is changing, the property is called **self inductance**, **L** When the e.m.f. is induced in a circuit by a change of flux due to current changing in an adjacent circuit, the property is called **mutual inductance**, **M**. The unit of inductance is the **henry**, **H**.

A circuit has an inductance of one henry when an e.m.f. of one volt is induced in it by a current changing at the rate of one ampere per second Induced e.m.f. in a coil of N turns,

$$E = -N \frac{\mathrm{d}\Phi}{\mathrm{d}t}$$
 volts

where $d\Phi$ is the change in flux in Webers, and dt is the time taken for the flux to change in seconds (i.e. $\frac{d\Phi}{dt}$ is the rate of change of flux).

Induced e.m.f. in a coil of inductance L henrys,

$$E = -L \frac{\mathrm{d}I}{\mathrm{d}t}$$
 volts

where d*I* is the change in current in amperes and dt is the time taken for the current to change in seconds (i.e. $\frac{dI}{dt}$ is the rate of change of current). The minus sign in each of the above two equations remind us of its direction (given by Lenz's law)

Problem 6. Determine the e.m.f. induced in a coil of 200 turns when there is a change of flux of 25 mWb linking with it in 50 ms.

Induced e.m.f.
$$E = -N \frac{d\Phi}{dt}$$

= $-(200) \left(\frac{25 \times 10^{-3}}{50 \times 10^{-3}}\right)$
= -100 volts

Problem 7. A flux of $400 \,\mu\text{Wb}$ passing through a 150-turn coil is reversed in 40 ms. Find the average e.m.f. induced.

Since the flux reverses, the flux changes from $+400 \,\mu\text{Wb}$ to $-400 \,\mu\text{Wb}$, a total change of flux of $800 \,\mu\text{Wb}$.

Induced e.m.f.
$$E = -N \frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$= -(150) \left(\frac{800 \times 10^{-6}}{40 \times 10^{-3}}\right)$$
$$= -\frac{150 \times 800 \times 10^{3}}{40 \times 10^{6}}$$

Hence, the average e.m.f. induced, E = -3 volts

Problem 8. Calculate the e.m.f. induced in a coil of inductance 12 H by a current changing at the rate of 4 A/s.

Induced e.m.f.
$$E = -L\frac{dI}{dt} = -(12)(4)$$

= -48 volts

Problem 9. An e.m.f. of 1.5 kV is induced in a coil when a current of 4 A collapses uniformly to zero in 8 ms. Determine the inductance of the coil.

Change in current,
$$dI = (4 - 0) = 4 \text{ A}$$
,
 $dt = 8 \text{ ms} = 8 \times 10^{-3} \text{ s}$.

$$\frac{dI}{dt} = \frac{4}{8 \times 10^{-3}} = \frac{4000}{8}$$

= 500 A/s

and

$$E = 1.5 \,\mathrm{kV} = 1500 \,\mathrm{V}$$

Since $|E| = L \frac{\mathrm{d}I}{\mathrm{d}t}$,

inductance,
$$L = \frac{|E|}{(dI/dt)} = \frac{1500}{500} = 3 \text{ H}$$

(Note that |E| means the 'magnitude of E' which disregards the minus sign)

Problem 10. An average e.m.f. of 40 V is induced in a coil of inductance 150 mH when a current of 6 A is reversed. Calculate the time taken for the current to reverse.

|E| = 40 V, L = 150 mH = 0.15 H and change in current, dI = 6 - (-6) = 12 A (since the current is reversed).

Since $|E| = \frac{\mathrm{d}I}{\mathrm{d}t}$,

time dt =
$$\frac{LdI}{|E|} = \frac{(0.15)(12)}{40}$$

= 0.045 s or 45 ms

Now try the following exercise

Exercise 42 Further problems on inductance

- 1 Find the e.m.f. induced in a coil of 200 turns when there is a change of flux of 30 mWblinking with it in 40 ms. [-150 V]
- 2 An e.m.f. of 25 V is induced in a coil of 300 turns when the flux linking with it changes by 12 mWb. Find the time, in milliseconds, in which the flux makes the change. [144 ms]
- 3 An ignition coil having 10 000 turns has an e.m.f. of 8 kV induced in it. What rate of change of flux is required for this to happen? [0.8 Wb/s]
- 4 A flux of 0.35 mWb passing through a 125turn coil is reversed in 25 ms. Find the magnitude of the average e.m.f. induced. [3.5 V]
- 5 Calculate the e.m.f. induced in a coil of inductance 6 H by a current changing at a rate of 15 A/s [-90 V]

9.4 Inductors

A component called an inductor is used when the property of inductance is required in a circuit. The basic form of an inductor is simply a coil of wire. Factors which affect the inductance of an inductor include:

- (i) the number of turns of wire the more turns the higher the inductance
- (ii) the cross-sectional area of the coil of wire the greater the cross-sectional area the higher the inductance
- (iii) the presence of a magnetic core when the coil is wound on an iron core the same current sets up a more concentrated magnetic field and the inductance is increased
- (iv) the way the turns are arranged a short thick coil of wire has a higher inductance than a long thin one.

Two examples of practical inductors are shown in Fig. 9.6, and the standard electrical circuit diagram symbols for air-cored and iron-cored inductors are shown in Fig. 9.7









Figure 9.7

An iron-cored inductor is often called a **choke** since, when used in a.c. circuits, it has a choking effect, limiting the current flowing through it.

Inductance is often undesirable in a circuit. To reduce inductance to a minimum the wire may be bent back on itself, as shown in Fig. 9.8, so that the magnetising effect of one conductor is neutralised by that of the adjacent conductor. The wire may be coiled around an insulator, as shown, without increasing the inductance. Standard resistors may be non-inductively wound in this manner.



Figure 9.8

9.5 Energy stored

An inductor possesses an ability to store energy. The energy stored, W, in the magnetic field of an inductor is given by:

$$W = \frac{1}{2}LI^2$$
 joules

Problem 11. An 8 H inductor has a current of 3 A flowing through it. How much energy is stored in the magnetic field of the inductor?

Energy stored,

$$W = \frac{1}{2}LI^2 = \frac{1}{2}(8) (3)^2 = 36$$
 joules

Now try the following exercise

Exercise 43 Further problems on energy stored

- 1 An inductor of 20 H has a current of 2.5 A flowing in it. Find the energy stored in the magnetic field of the inductor. [62.5 J]
- 2 Calculate the value of the energy stored when a current of 30 mA is flowing in a coil of inductance 400 mH [0.18 mJ]
- 3 The energy stored in the magnetic field of an inductor is 80 J when the current flowing in the inductor is 2 A. Calculate the inductance of the coil. [40 H]

9.6 Inductance of a coil

If a current changing from 0 to *I* amperes, produces a flux change from 0 to Φ webers, then dI = I and $d\Phi = \Phi$. Then, from section 9.3,

induced e.m.f.
$$E = \frac{N\Phi}{t} = \frac{LI}{t}$$

from which, inductance of coil,

$$L = \frac{N\Phi}{I}$$
 henrys

Problem 12. Calculate the coil inductance when a current of 4 A in a coil of 800 turns produces a flux of 5 mWb linking with the coil.

For a coil, inductance

$$L = \frac{N\Phi}{I} = \frac{(800)(5 \times 10^{-3})}{4} = 1 \,\mathrm{H}$$

Problem 13. A flux of 25 mWb links with a 1500 turn coil when a current of 3 A passes through the coil. Calculate (a) the inductance of the coil, (b) the energy stored in the magnetic field, and (c) the average e.m.f. induced if the current falls to zero in 150 ms.

(a) Inductance,

$$L = \frac{N\Phi}{I} = \frac{(1500)(25 \times 10^{-3})}{3} = 12.5 \,\mathrm{H}$$

(b) Energy stored,

$$V = \frac{1}{2}LI^2 = \frac{1}{2}(12.5)(3)^2 = 56.25 \text{ J}$$

(c) Induced emf,

$$E = -L\frac{dI}{dt} = -(12.5)\left(\frac{3-0}{150\times10^{-3}}\right)$$
$$= -250 \,\mathrm{V}$$

(Alternatively,

$$E = -N \frac{d\Phi}{dt}$$

= -(1500) $\left(\frac{25 \times 10^{-3}}{150 \times 10^{-3}}\right)$
= -250 V

since if the current falls to zero so does the flux)

Problem 14. When a current of 1.5 A flows in a coil the flux linking with the coil is 90μ Wb. If the coil inductance is 0.60 H, calculate the number of turns of the coil.

For a coil,
$$L = \frac{N\Phi}{I}$$
. Thus
 $N = \frac{LI}{\Phi} = \frac{(0.6)(1.5)}{90 \times 10^{-6}} = 10\,000 \,\text{turns}$

Problem 15. A 750 turn coil of inductance 3 H carries a current of 2 A. Calculate the flux linking the coil and the e.m.f. induced in the coil when the current collapses to zero in 20 ms.

Coil inductance, $L = \frac{N\Phi}{I}$ from which, flux

$$\Phi = \frac{LI}{N} = \frac{(3)(2)}{750} = 8 \times 10^{-3} = 8 \,\mathrm{mWb}$$

Induced e.m.f.

$$E = -L\frac{dI}{dt} = -(3)\left(\frac{2-0}{20 \times 10^{-3}}\right)$$

= -300 V

(Alternatively,

$$E = -N \frac{d\Phi}{dt} = -(750) \left(\frac{8 \times 10^{-3}}{20 \times 10^{-3}}\right)$$
$$= -300 \text{ V})$$

Now try the following exercise

Exercise 44 Further problems on the inductance of a coil

1 A flux of 30 mWb links with a 1200 turn coil when a current of 5 A is passing through the coil. Calculate (a) the inductance of the coil, (b) the energy stored in the magnetic field, and (c) the average e.m.f. induced if the current is reduced to zero in 0.20 s [(a) 7.2 H (b) 90 J (c) 180 V]

2 An e.m.f. of 2 kV is induced in a coil when a current of 5 A collapses uniformly to zero in 10 ms. Determine the inductance of the coil. [4 H]

- 3 An average e.m.f. of 60 V is induced in a coil of inductance 160 mH when a current of 7.5 A is reversed. Calculate the time taken for the current to reverse. [40 ms]
- 4 A coil of 2500 turns has a flux of 10 mWb linking with it when carrying a current of 2 A. Calculate the coil inductance and the e.m.f. induced in the coil when the current collapses to zero in 20 ms. [12.5 H, 1.25 kV]

- 5 Calculate the coil inductance when a current of 5 A in a coil of 1000 turns produces a flux of 8 mWb linking with the coil. [1.6 H]
- 6 A coil is wound with 600 turns and has a self inductance of 2.5 H. What current must flow to set up a flux of 20 mWb ? [4.8 A]
- 7 When a current of 2 A flows in a coil, the flux linking with the coil is 80μ Wb. If the coil inductance is 0.5 H, calculate the number of turns of the coil. [12 500]
- 8 A coil of 1200 turns has a flux of 15 mWb linking with it when carrying a current of 4 A. Calculate the coil inductance and the e.m.f. induced in the coil when the current collapses to zero in 25 ms [4.5 H, 720 V]
- 9 A coil has 300 turns and an inductance of 4.5 mH. How many turns would be needed to produce a 0.72 mH coil assuming the same core is used ? [48 turns]
- 10 A steady current of 5 A when flowing in a coil of 1000 turns produces a magnetic flux of 500μ Wb. Calculate the inductance of the coil. The current of 5 A is then reversed in 12.5 ms. Calculate the e.m.f. induced in the coil. [0.1 H, 80 V]

9.7 Mutual inductance

Mutually induced e.m.f. in the second coil,

$$E_2 = -M \frac{\mathrm{d}I_1}{\mathrm{d}t}$$
 volts

where *M* is the **mutual inductance** between two coils, in henrys, and (dI_1/dt) is the rate of change of current in the first coil.

The phenomenon of mutual inductance is used in **transformers** (see chapter 21, page 303)

Problem 16. Calculate the mutual inductance between two coils when a current changing at 200 A/s in one coil induces an e.m.f. of 1.5 V in the other.

Induced e.m.f. $|E_2| = M dI_1/dt$, i.e. 1.5 = M(200). Thus **mutual inductance**,

$$M = \frac{1.5}{200} = 0.0075 \,\mathrm{H} \text{ or } 7.5 \,\mathrm{mH}$$

Problem 17. The mutual inductance between two coils is 18 mH. Calculate the steady rate of change of current in one coil to induce an e.m.f. of 0.72 V in the other.

Induced e.m.f. $|E_2| = M \frac{dI_1}{dt}$ Hence rate of change of current,

$$\frac{\mathrm{d}I_1}{\mathrm{d}t} = \frac{|E_2|}{M} = \frac{0.72}{0.018} = 40 \,\mathrm{A/s}$$

Problem 18. Two coils have a mutual inductance of 0.2 H. If the current in one coil is changed from 10 A to 4 A in 10 ms, calculate (a) the average induced e.m.f. in the second coil, (b) the change of flux linked with the second coil if it is wound with 500 turns.

(a) Induced e.m.f.

$$|E_2| = -M \frac{dI_1}{dt}$$

= -(0.2) $\left(\frac{10-4}{10 \times 10^{-3}}\right) = -120 \text{ V}$

(b) Induced e.m.f.

$$|E_2| = N \frac{\mathrm{d}\Phi}{\mathrm{d}t}$$
, hence $\mathrm{d}\Phi = \frac{|E_2|\mathrm{d}t}{N}$

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Thus the change of flux,

$$\mathrm{d}\Phi = \frac{(120)(10 \times 10^{-3})}{500} = 2.4\,\mathrm{mWb}$$

Now try the following exercises

Exercise 45 Further problems on mutual inductance

1 The mutual inductance between two coils is 150 mH. Find the magnitude of the e.m.f.

induced in one coil when the current in the other is increasing at a rate of 30 A/s.

[4.5 V]

- 2 Determine the mutual inductance between two coils when a current changing at 50 A/s in one coil induces an e.m.f. of 80 mV in the other. [1.6 mH]
- 3 Two coils have a mutual inductance of 0.75 H. Calculate the magnitude of the e.m.f. induced in one coil when a current of 2.5 A in the other coil is reversed in 15 ms [250 V]
- 4 The mutual inductance between two coils is 240 mH. If the current in one coil changes from 15 A to 6 A in 12 ms, calculate (a) the average e.m.f. induced in the other coil, (b) the change of flux linked with the other coil if it is wound with 400 turns.

[(a) -180 V (b) 5.4 mWb]

5 A mutual inductance of 0.06 H exists between two coils. If a current of 6A in one coil is reversed in 0.8 s calculate (a) the average e.m.f. induced in the other coil, (b) the number of turns on the other coil if the flux change linking with the other coil is 5 mWb

[(a) -0.9 V (b) 144]

Exercise 46 Short answer questions on electromagnetic induction

- 1 What is electromagnetic induction?
- 2 State Faraday's laws of electromagnetic induction
- 3 State Lenz's law
- 4 Explain briefly the principle of the generator
- 5 The direction of an induced e.m.f. in a generator may be determined using Fleming's rule
- 6 The e.m.f. *E* induced in a moving conductor may be calculated using the formula E = Blv. Name the quantities represented and their units
- 7 What is self-inductance? State its symbol
- 8 State and define the unit of inductance
- 9 When a circuit has an inductance L and the current changes at a rate of (di/dt) then the induced e.m.f. E is given by $E = \dots$ volts

- 10 If a current of *I* amperes flowing in a coil of *N* turns produces a flux of Φ webers, the coil inductance *L* is given by $L = \dots$ henrys
- 11 The energy W stored by an inductor is given by $W = \dots$ joules
- 12 What is mutual inductance ? State its symbol
- 13 The mutual inductance between two coils is M. The e.m.f. E_2 induced in one coil by the current changing at (dI_1/dt) in the other is given by $E_2 = \dots$ volts

Exercise 47 Multi-choice questions on electromagnetic induction (Answers on page 375)

- 1 A current changing at a rate of 5 A/s in a coil of inductance 5 H induces an e.m.f. of:
 - (a) 25 V in the same direction as the applied voltage
 - (b) 1 V in the same direction as the applied voltage
 - (c) 25 V in the opposite direction to the applied voltage
 - (d) 1 V in the opposite direction to the applied voltage
- 2 A bar magnet is moved at a steady speed of 1.0 m/s towards a coil of wire which is connected to a centre-zero galvanometer. The magnet is now withdrawn along the same path at 0.5 m/s. The deflection of the galvanometer is in the:
 - (a) same direction as previously, with the magnitude of the deflection doubled
 - (b) opposite direction as previously, with the magnitude of the deflection halved
 - (c) same direction as previously, with the magnitude of the deflection halved
 - (d) opposite direction as previously, with the magnitude of the deflection doubled
- 3 When a magnetic flux of 10 Wb links with a circuit of 20 turns in 2 s, the induced e.m.f. is:
 (a) 1 V
 (b) 4 V
 (c) 100 V
 (d) 400 V
- 4 A current of 10 A in a coil of 1000 turns produces a flux of 10 mWb linking with the coil. The coil inductance is: (a) 10^6 H (b) 1 H
 - (c) $1 \mu H$ (d) 1 m H
- 5 An e.m.f. of 1 V is induced in a conductor moving at 10 cm/s in a magnetic field of

0.5 T. The effective length of the conductor in the magnetic field is:

- (a) 20 cm (b) 5 m
- (c) 20 m (d) 50 m
- 6 Which of the following is false ?
 - (a) Fleming's left-hand rule or Lenz's law may be used to determine the direction of an induced e.m.f.
 - (b) An induced e.m.f. is set up whenever the magnetic field linking that circuit changes
 - (c) The direction of an induced e.m.f. is always such as to oppose the effect producing it
 - (d) The induced e.m.f. in any circuit is proportional to the rate of change of the magnetic flux linking the circuit
- 7 The effect of inductance occurs in an electrical circuit when:
 - (a) the resistance is changing
 - (b) the flux is changing
 - (c) the current is changing
- 8 Which of the following statements is false? The inductance of an inductor increases:
 - (a) with a short, thick coil
 - (b) when wound on an iron core

- (c) as the number of turns increases
- (d) as the cross-sectional area of the coil decreases
- 9 The mutual inductance between two coils, when a current changing at 20 A/s in one coil induces an e.m.f. of 10 mV in the other, is:
 (a) 0.5 H
 (b) 200 mH
 (c) 0.5 mH
 (d) 2 H
- 10 A strong permanent magnet is plunged into a coil and left in the coil. What is the effect produced on the coil after a short time?(a) There is no effect
 - (b) The insulation of the coil burns out
 - (c) A high voltage is induced
 - (d) The coil winding becomes hot
- 11 Self-inductance occurs when:
 - (a) the current is changing
 - (b) the circuit is changing
 - (c) the flux is changing
 - (d) the resistance is changing
- 12 Faraday's laws of electromagnetic induction are related to:
 - (a) the e.m.f. of a chemical cell
 - (b) the e.m.f. of a generator
 - (c) the current flowing in a conductor
 - (d) the strength of a magnetic field